J = particle flux, particles/(cm^2-s)
k = Boltzmann constant, erg/°K
$K_f = \left\{ \int_{\delta_2}^{\delta_1} \frac{\exp[\phi(h)/kT]}{D(h)} dh \right\}^{-1}$, cm/s
$K_r = K_f/I_2$, s^{-1}
m = local particle mobility, (cm/s)/dyne
M = mass of a single particle, g
n = ionic strength of bulk solution, ions/cm^3*
n_2 = number of adsorbed particles, particles/cm^2
n_{2i} = initial value of n_2, particles/cm^2
n_{2f} = final value of n_2, particles/cm^2
T = absolute temperature, °K
U = approach velocity, cm/s
y = distance measured normal to collector, cm

Greek Letters

$\alpha = [(0.170/D(\infty))(\omega^3\rho_f/\mu)^{1/2}]^{1/3}$, cm^{-1}
$\beta = 1 - 0.5 \ln(2aU\rho_f/\mu)$
$\gamma = |d^2\phi/dh^2|$, erg/cm^2
Γ = Gamma function
δ_o = thickness of interaction boundary layer, cm
δ_1 = smallest h in the interval $h_{max} < h < h_{mn1}$ for which a Boltzmann distribution of particles may be assumed, cm
δ_2 = largest h in the interval $h_{mn2} < h < h_{max}$ for which a Boltzmann distribution of particles may be assumed, cm
ϵ = fluid dielectric constant
$\kappa = (8\pi ne^2/\epsilon kT)^{1/2}$, cm^{-1}
μ = fluid viscosity, g/(cm-s)
ρ_f = fluid density, g/cm^3
σ = collision diameter, cm
σ' = electrostatic surface charge density, statcoul*
ϕ = total potential energy of interaction, erg
ϕ_6 = van der Waals energy of interaction, erg
ϕ_{12} = Born energy of interaction, erg
ϕ_{DL} = double-layer energy of interaction, erg
ψ_{Si} = electrostatic potential of surface i, statvolt*
ω = disk rotation speed, rad/s

Subscripts

max = evaluated at the maximum of $\phi(h)$
mn1 = evaluated at the secondary minimum of $\phi(h)$
mn2 = evaluated at the primary minimum of $\phi(h)$

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* A consistent set of units has been specified above for each symbol, although more convenient units are sometimes employed in the text (for example, mV instead of statvolts).

Manuscript received July 8, 1975; revision received November 13 and accepted November 14, 1975.

Constitutive Equation for Vapor Drift Velocity in Two-Phase Annular Flow

A constitutive equation for a vapor drift velocity which specifies the relative motion between phases in the drift flux model is developed for two-phase annular flows. The constitutive equation is derived by taking into account the interfacial geometry, the body force field, and the interfacial momentum transfer, since these macroscopic effects govern the two-phase diffusions. A comparison of the model with three sets of experimental data obtained over a wide range of flow parameters shows a satisfactory agreement.

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SCOPE

A general transient two-phase flow problem can be formulated by using a two-fluid model or a drift flux model. In the two-fluid model, each phase is considered separately with two sets of conservation equations of mass, momentum, and energy. However, an introduction of two momentum equations in a formulation as in the case of two-fluid model presents considerable difficulties because of mathematical complications and because of uncertainties in specifying interfacial interaction terms between two phases (Ishii, 1975). Numerical instabilities caused by improper choice of interfacial interaction terms in the phase momentum equations are quite common, and therefore very careful studies on the interfacial constitutive equations are required in the two-fluid model formulation. For example, it has been suggested (Reocreux, 1974) that the interaction terms should include first-order time and spatial derivatives under certain conditions. These difficulties associated with a two-fluid model can be significantly reduced by formulating two-phase problems in terms of the drift flux model (Zuber, 1967) in which the motion of the whole mixture is expressed by the mixture momentum equation, and the relative motion between phases is taken into account by a kinematic constitutive equation. The use of the drift flux model is appropriate when the dynamics of two phases are strongly coupled. Consequently, in the drift flux model, the dynamic interactions between phases of the two-fluid model are replaced by a kinematic relation between the two velocity fields.

Because of its simplicity and applicability to a wide range of two-phase flow systems of practical interest, the drift flux model is of considerable importance. In particular, the model is useful for thermohydraulic analyses of light-water reactors and liquid-metal-cooled fast breeder reactors under transient and accident conditions.

In the drift flux model of Zuber et al. (1967), the velocity fields are expressed in terms of the mixture center-of-mass velocity and the drift velocity of the vapor phase, which is the vapor velocity with respect to the volume center of the mixture. In order to close the system of equations, it is necessary to specify this vapor drift velocity by a constitutive equation, since in the drift flux model the motion of the fluid is expressed by the mixture momentum equation alone. This is analogous to the use of the Fick's law of diffusion for single-phase, two-component systems in which the diffusion of components is governed by the concentration gradient, and the relative motion is expressed through a diffusion coefficient. The purpose of the paper is to develop the kinematic constitutive equation for two-phase annular flow in which the relative motion between phases is governed by macroscopic structure of the flows and to compare the present model with various experimental data.

CONCLUSIONS AND SIGNIFICANCE

In the drift flux model, it is necessary to specify the relative motion between phases by a constitutive equation in order to take into account the diffusions of one phase with respect to another. The mixture momentum equation together with this kinematic constitutive equation specify the motions of each phase.

In view of the practical importance of the drift flux model for two-phase flow analyses in general and in analyses of nuclear-reactor accidents in particular, the kinematic constitutive equation for the vapor drift velocity for two-phase annular flows has been developed. The present model can be applied to an annular flow with liquid film either in laminar or turbulent flow and allows for the effect of gravity and void fraction. Consequently, it can be used for upward and downward concurrent annular flows as well as for countercurrent separated flows. The constitutive equation for the drift velocity developed here extends the range of applicability of the drift flux model (Zuber, 1967) to the separated two-phase flow regime.

The constitutive equation obtained in this paper is cast into two different forms, one useful for analyzing steady state flows and the other for transient flows. The predicted vapor drift velocity from this study was compared to about 350 data points from various experiments. Although the data used in the comparison included those taken in the drop-annular flow regimes with moderate entrainment, the theoretical predictions are within $\pm 30\%$ of the measured values. However, when the amount of liquid entrainment was large, the present correlation overpredicted the vapor drift velocity.

DRIFT FLUX MODEL

In two-phase flows, there is always some relative motion of one phase with respect to the other. Therefore, a two-phase flow problem should be formulated in terms of two velocity fields. According to the degree of the dynamic coupling between the phases, two-phase flow problems can be formulated in terms of a two-fluid model or a drift flux model (Zuber, 1967; Kocamustafaogullari, 1971; Ishii, 1975). The two-fluid model is formulated in terms of two sets of conservation equations governing the balance of mass, momentum, and energy of each phase. However, since the macroscopic fields of one phase are not independent of the other phase, interaction terms which couple the transports of mass, momentum, and energy of each phase across the interfaces should appear in the field equations. On the other hand, the drift flux model (or mixture model) is formulated by considering the mixture as a whole, rather than two phases separately. Therefore, the drift flux model requires only four field equations, namely, equations of continuity, momentum, and energy for the mixture, and the continuity equation for one of the phases (Zuber, 1967).

In the two-fluid model formulation, the transport processes of each phase are expressed by their own balance equations; therefore, it is expected that the model can predict more detailed changes and phase interactions than the drift flux model. However, this also implies that the two-fluid model is considerably more complicated not only because of the increased number of field equations and the unknown, but also because of the several necessary constitutive equations. Furthermore, these constitutive equations should be formulated fairly accurately to offer any meaningful advantage of the two-fluid model over the drift flux model. This is particularly true in regard to phase interaction terms which specify the exchange of mass, momentum, and energy at the interfaces. However, accurate

constitutive equations for interaction terms under transient conditions are largely unknown. Moreover, the use of existing inaccurate constitutive equations can result in numerical instabilities, since the two-fluid model is inherently unstable owing to the Kelvin-Helmholtz instability unless a proper stabilization mechanism is built into the model through constitutive equations.

It is evident that the drift flux model is simpler than the two-fluid model. However, the former requires some drastic constitutive assumptions, since it has only four field equations in contrast to six field equations in the two-fluid model. It is therefore natural that some of the characteristics of two-phase flow will be lost. The simplification introduced by using only four field equations rather than six makes the use of the drift flux model a very attractive and powerful technique for analyzing a number of engineering problems. In particular, the drift flux model is useful when the information required is the response of the total mixture and not that of each constituent phase separately. For example, in the dynamic analyses of two-phase flow systems, only the response of the total system is desired. In this connection, the drift flux model has already been applied to the thermohydraulic flow instability problem in boiling channels (Ishii and Zuber, 1970; Saha, 1974).

The four field equations in the drift flux model are the result of the elimination of one energy and one momentum equation from the original six field equations of the two-fluid model. Therefore, the relative motion and energy differences should be expressed by additional constitutive equations. These two effects inherent to the two-phase flows are taken into account by using a continuity equation for one of the phases and supplementing it with kinematic and phase change constitutive equations.

The continuity equation of a phase denoted as k^{th} component is given by

$$\frac{\partial \alpha_k \rho_k}{\partial t} + \frac{\partial}{\partial z}(\alpha_k \rho_k v_k) = \Gamma_k \quad (k = g \text{ or } f) \tag{1}$$

where Γ_k denotes the mass source from the interfaces. The total mass balance across the interfaces requires that

$$\Gamma_g + \Gamma_f = 0 \tag{2}$$

Therefore, by adding the two-phase continuity equations, we obtain the mixture continuity equation given by

$$\frac{\partial \rho_m}{\partial t} + \frac{\partial \rho_m v_m}{\partial z} = 0 \tag{3}$$

where the mixture density is defined by

$$\rho_m \equiv \alpha \rho_g + (1 - \alpha)\rho_f \tag{4}$$

and the mixture center-of-mass velocity by

$$v_m \equiv [\alpha \rho_g v_g + (1 - \alpha)\rho_f v_f]/\rho_m \tag{5}$$

On the other hand, the velocity of the center of volume of the mixture, that is, the total volumetric flux, is defined by

$$j \equiv \alpha v_g + (1 - \alpha) v_f \tag{6}$$

The vapor drift velocity V_{gj} is the velocity of the vapor with respect to the volume center of the mixture; therefore, we have

$$V_{gj} \equiv v_g - j = (1 - \alpha) v_r \tag{7}$$

where v_r is the relative velocity between phases. The advantage of using the vapor drift velocity is apparent for dispersed two-phase systems, since it can be easily related to a rise velocity of bubbles or a terminal velocity of particles or drops. For separated two-phase flows, this advantage may not exist; however, whether we should use a constitutive equation for V_{gj} or v_r or any other velocities is immaterial, since these can be related by definitions. From the definitions of various velocity fields, it is straightforward to show that

$$v_g = v_m + \frac{\rho_f}{\rho_m} V_{gj} \tag{8}$$

By substituting Equation (8) into the vapor continuity equation, Equation (1), we obtain

$$\frac{\partial \alpha \rho_g}{\partial t} + \frac{\partial}{\partial z}(\alpha \rho_g v_m) = \Gamma_g - \frac{\partial}{\partial z}\left[\frac{\alpha \rho_g \rho_f}{\rho_m} V_{gj}\right] \tag{9}$$

Since the convective term in the left-hand side of the above equation is expressed by the mixture velocity v_m, an additional diffusion term appears on the right-hand side of Equation (9). Thus, in order to take into account the effects of the relative motion between phases in the drift flux model, the vapor drift velocity should be specified. This kinematic constitutive equation may be formulated in a functional form as

$$V_{gj} = V_{gj}(\alpha, p, g, v_m, \text{etc.}) \tag{10}$$

This constitutive equation replaces one of the phase momentum equations of the two-fluid model formulation. In order to take into account the mass transfer across the interfaces, a constitutive equation for Γ_g should also be given. In a functional form, this phase change constitutive equation may be written as

$$\Gamma_g = \Gamma_g\left(\alpha, p, v_m, \frac{\partial p}{\partial t}, \text{etc.}\right) \tag{11}$$

It may be noted that the vapor drift velocity in the drift flux model plays a role similar to that of the diffusion coefficient in a single-phase, two-component system. However, the application of a diffusion coefficient is useful only when the relative motion between components or phases is due to a concentration gradient and can be expressed by a linear constitutive law. For general two-phase flow systems, it is expected that the Fick's law of diffusion may not hold, since in this case the interfacial geometry, the body force field, and the interfacial momentum transfer are the factors governing the relative motion of phases. In other words, the diffusion of phases in two-phase systems is macroscopic, whereas in single-phase, two-component systems, it is due to the microscopic molecular diffusion.

It is therefore expected that the constitutive equation for the vapor drift velocity strongly depends on the two-phase flow regimes, since the momentum transfer between the phases is governed by the geometry of the interfaces as well as by the interfacial area concentration. The constitutive equation for the vapor drift velocity for bubbly and slug flow regimes has been obtained by Zuber (1964), Zuber and Findley (1965), and Zuber et al. (1967) by balancing the gravity force with the drag force. Staub (1966) has attempted to obtain the drift velocity for an annular flow regime. His analysis is based on a linear velocity profile in the liquid film and contains an empirical constant. The linear velocity profile is valid only when the film flow is laminar and the gravitational effect is negligible. It has been demonstrated in the present analysis that Staub's empirical constant is actually a function of a void fraction.

The purpose of the paper is to present a more general expression for the drift velocity for an annular flow by taking into account the effect of gravity, interfacial shear stress with its dependence on interfacial roughness, and flow regimes in the liquid film.

A particularly important application of the present model is in the area of computer code developments for

transient thermohydraulic analyses of two-phase flows, since one of the flow regimes to be expected in systems is annular flow. Another important application is in accident analyses of liquid-metal fast breeder reactors, for which the most likely flow regime encountered in sodium boiling is also annular flow.

ANALYSIS

Inasmuch as the vapor drift velocity in the drift flux model is a substitute for one of the phase momentum equations, a natural starting point for its derivation is the momentum equations for two components. Assuming steady state adiabatic two-phase annular flow with constant single-phase properties, we have the following one-dimensional momentum equations for each phase:

$$-\left(\frac{dp}{dz}+\rho_g g\right)=\frac{\tau_i P_i}{\alpha A} \tag{12}$$

and

$$-\left(\frac{dp}{dz}+\rho_f g\right)=\frac{\tau_{wf}P_{wf}}{A(1-\alpha)}-\frac{\tau_i P_i}{A(1-\alpha)} \tag{13}$$

We define the hydraulic diameter D and the ratio of the wetted perimeters by

$$D\equiv 4A/P_{wf};\quad \xi\equiv P_i/P_{wf} \tag{14}$$

By assuming that the film thickness δ is small compared with the hydraulic diameter, we have

$$\frac{4\delta}{D}\simeq(1-\alpha) \tag{15}$$

We note here that for an annular flow in a pipe, ξ reduces to $\sqrt{\alpha}$. From the analyses of thin films, the wall shear stress both for laminar and turbulent flows can be given approximately by

$$\tau_{wf}=\frac{f_{wf}\rho_f v_f|v_f|}{2}-\frac{1}{3}\Delta\rho g\delta \tag{16}$$

Here the gravity correction term on the right-hand side of Equation (16), which arises naturally for a laminar film flow, has been assumed to be the same for a turbulent flow as well. The friction factor (Hewitt and Hall-Taylor, 1970) is then given by

$$f_{wf}=\begin{cases}16/Re_f & \text{(laminar flow)}\\ 0.0791\,Re_f^{-0.25} & \text{(turbulent flow)}\end{cases} \tag{17}$$

where the film Reynolds number is given by

$$Re_f=\frac{4\rho_f|v_f|\delta}{\mu_f}\simeq\frac{\rho_f|j_f|D}{\mu_f} \tag{18}$$

Here μ_f is the liquid viscosity and j_f is the liquid volumetric flux; that is, $j_f=(1-\alpha)v_f$.

The interfacial shear stress τ_i can be expressed as

$$\tau_i=\frac{f_i\rho_g v_r^2}{2} \tag{19}$$

where v_r is the relative velocity between the phases. The interfacial friction factor f_i can be given, for example, from Wallis' correlation (1970) as

$$f_i=\pm 0.005[1+\epsilon]\begin{cases}+ & \text{for } V_{gj}\geq 0\\ - & \text{for } V_{gj}<0\end{cases} \tag{20}$$

with the roughness parameter given by

$$\epsilon=300\,\delta/D\simeq 75\,(1-\alpha) \tag{21}$$

By definition, the vapor drift velocity is related to v_r through Equation (7). Eliminating dp/dz between Equations (12) and (13), and using Equations (14) to (19) in the resultant equation, we obtain the vapor drift velocity for a laminar flow in the liquid film:

$$V_{gj}^2=\frac{16\alpha}{\rho_g f_i\xi}\left[\frac{\mu_f j_f}{D}+\frac{\Delta\rho g D(1-\alpha)^3}{48}\right] \tag{22}$$

In contrast to the above expression, the semiempirical correlation of Staub (1966) is given by $V_{gj}=23\{\mu_f j_f/(\rho_g D)\}^{1/2}$, where the empirical constant 23 has been obtained from a very limited number of experimental data. By comparing his correlation to the present model for a separated flow with a laminar liquid film, it is evident that the important contribution from the gravity force has been neglected in Staub (1966), and the empirical constant is actually a function of void fraction. For an annular flow in a pipe with $\xi=\sqrt{\alpha}$ and $0.8<\alpha<1$, this empirical constant varies from a value of 13 to a value of 50. This shows that the Staub's correlation is an overly simplified form of Equation (22) based only on limited experimental data.

The drift velocity for a separated flow with turbulent film flow is obtained similarly as

$$V_{gj}^2=\frac{\alpha(1-\alpha)^3 D}{\xi f_i\rho_g}\left[\left(\frac{0.0791}{Re_f^{0.25}}\right)\frac{\rho_f j_f|j_f|}{D(1-\alpha)^3}+\frac{1}{3}\Delta\rho g\right] \tag{23}$$

We note here that V_{gj} takes a negative root of Equation (22) or (23) if the value of the terms in the square bracket is negative.

A simpler form of Equation (23) can be obtained by use of a constant turbulent, wall friction factor defined as

$$f_{wf}=\begin{cases}16/Re_f & \text{if } Re_f\leq 3\,200\\ 0.005 & \text{if } Re_f>3\,200\end{cases} \tag{24}$$

which was also used by Wallis (1970) in his simple annular flow theory. Then the transition liquid volumetric flux at the laminar turbulent transition point is given by

$$j_{ftr}=\frac{3\,200\,\mu_f}{\rho_f D} \tag{25}$$

The vapor drift velocity for a separated flow with a turbulent film flow becomes

$$V_{gj}^2=\frac{\alpha}{\xi}\,\frac{(1-\alpha)^3 D}{f_i\rho_g}\left[\frac{0.005\,\rho_f j_f|j_f|}{D(1-\alpha)^3}+\frac{1}{3}\Delta\rho g\right] \tag{26}$$

The vapor drift velocity V_{gj} in the form as expressed by Equations (22) and (23) or (26) is convenient for use in analyzing steady state adiabatic or thermal equilibrium flows, since in these cases the local value of j_f can be easily predicted. However, in a general drift flux model formulation, V_{gj} should be expressed in terms of the mixture velocity v_m rather than j_f, as it is the velocity field which is used in the description of the drift flux model (Zuber, 1967).

Drift Velocity as a Function of Mixture Velocity

From the definitions of mixture velocity; [$v_m=\{(1-\alpha)\rho_f v_f+\alpha\rho_g v_g\}/\rho_m$ and V_{gj} as given by Equation (7)], the liquid volumetric flux can be expressed in terms of the mixture velocity and the vapor drift velocity as

$$j_f=(1-\alpha)v_m-\frac{\alpha\rho_g}{\rho_m}V_{gj} \tag{27}$$

Substituting Equation (27) into Equation (22), we obtain for a laminar film

$$V_{gj} = \frac{8\mu_f\alpha^2}{\rho_m D f_i \xi}\left\{-1 + \left(1 + \frac{f_i D \rho_m^2 (1-\alpha)\xi}{4\mu_f \alpha^3 \rho_g}\left[v_m + \frac{\Delta\rho g D^2 (1-\alpha)^2}{48\mu_f}\right]\right)^{1/2}\right\} \quad (28)$$

Thus, in view of Equations (25) and (27), the above expression is valid for laminar film flow in concurrent or countercurrent flow situations with V_{gj} in the range

$$\frac{(1-\alpha)\rho_m v_m - \rho_m j_{ftr}}{\alpha\rho_g} \leqq V_{gj} \leqq \frac{(1-\alpha)\rho_m v_m + \rho_m j_{ftr}}{\alpha\rho_g} \quad (29)$$

By adopting the sign convention such that for $v_m \geqq -\Delta\rho g D^2(1-\alpha)^2/48\mu_f$, f_i has a positive sign and for $v_m < -\Delta\rho g D^2(1-\alpha)^2/48\mu_f$, f_i has a negative sign, Equation (28) is simultaneously valid for positive values of V_{gj} as well as for negative values of V_{gj}. The form of the solution of Equation (28) as a function of v_m keeping α constant is given in Figure 1. As can be seen from this figure, V_{gj} for laminar flow is bounded in the range given by Equation (29). For $V_{gj} \leqq (1-\alpha)\rho_m v_m/\alpha\rho_g$, the flow is concurrent upward; for $V_{gj} > (1-\alpha)\rho_m v_m/\alpha\rho_g$, the direction of the liquid flow is downward. The case $V_{gj} = 0$ corresponds to the free falling film without interfacial shear.

To obtain V_{gj} as a function of v_m for turbulent film flow, we substitute Equation (27) into Equation (26). Then, by solving for V_{gj} for the case with the upward liquid flow given by

$$v_m \geqq \sqrt{cb^2/a} = \left\{\frac{\Delta\rho g D(1-\alpha)^3\alpha^3\rho_g}{3\rho_m^2 f_i \xi}\right\}^{1/2}$$

we have

$$V_{gj} = \begin{cases} \dfrac{-bv_m + [av_m^2 + (a-b^2)c]^{1/2}}{(a-b^2)} & \text{if } a - b^2 \neq 0 \\ (v_m^2 + c)/2bv_m & \text{if } a - b^2 = 0 \end{cases} \quad (30)$$

Here, we have introduced for convenience

$$a \equiv \frac{f_i \xi \rho_g}{0.005\,\alpha\rho_f(1-\alpha)^2}$$

$$b \equiv \frac{\alpha\rho_g}{\rho_m(1-\alpha)} \quad (31)$$

$$c \equiv \frac{\Delta\rho g D(1-\alpha)^3}{0.015\,\rho_f}$$

However, for v_m lying in the range

$$-\sqrt{\frac{\Delta\rho g D(1-\alpha)^3}{0.015\,\rho_f}} = -\sqrt{c} < v_m < \sqrt{cb^2/a}$$

we have

$$V_{gj} = \frac{-bv_m + [-av_m^2 + (a+b^2)c]^{1/2}}{a+b^2} \quad (32)$$

Here the lower boundary for v_m corresponds to zero interfacial shear stress, whereas the upper limit is the point of the flow reversal. In the range of v_m given by $v_m \leqq -\sqrt{c}$, we have

$$V_{gj} = \frac{-bv_m + [av_m^2 - c(a-b^2)]^{1/2}}{a-b^2} \quad (33)$$

which applies to the concurrent downward flow.

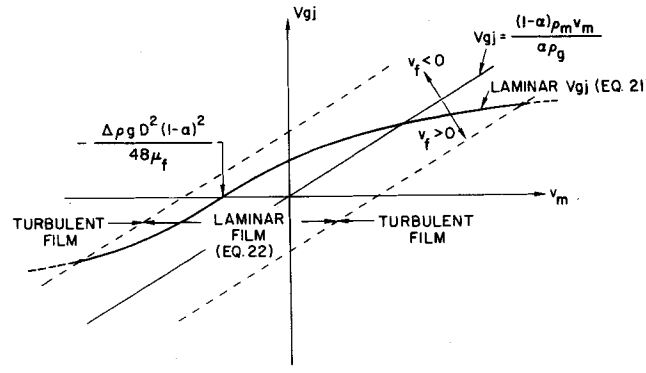

Fig. 1. Laminar film V_{gj} at constant void fraction.

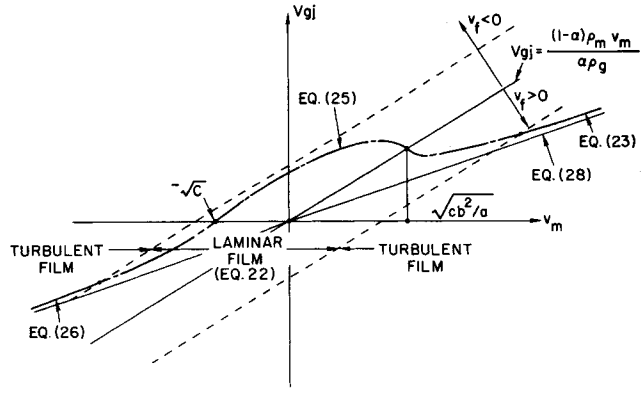

Fig. 2. Turbulent film V_{gj} at constant void fraction.

The above solutions for V_{gj} can be applied only if the turbulent criterion given below is satisfied:

$$\text{or} \quad \begin{aligned} V_{gj} &\leqq [(1-\alpha)\rho_m v_m - \rho_m j_{ftr}]/\alpha\rho_g \\ V_{gj} &\geqq [(1-\alpha)\rho_m v_m + \rho_m j_{ftr}]/\alpha\rho_g \end{aligned} \quad (34)$$

The solution of Equations (30), (32), and (33) for a fixed value of α is shown in Figure 2, where V_{gj} is plotted against v_m. In the figure, the laminar-turbulent transition occurs at the hyperbolic part of the solution. However, depending on the parameters, it is possible that the elliptic part, Equation (32), could occur in the turbulent regime, that is, in the range given by Equation (34).

If the absolute value of the mixture velocity is large, such that the flow is essentially concurrent and the gravity effect is small, then the turbulent solution can be approximated by the simple form

$$V_{gj} = \frac{(1-\alpha)v_m}{\dfrac{\alpha\rho_g}{\rho_m} + \left\{\dfrac{\xi\rho_g[1+75(1-\alpha)]}{\alpha\rho_f}\right\}^{1/2}} \quad (35)$$

As it can be seen in Figure 2, the expression given by Equation (35) is an asymptote to the more exact solution given by Equations (30) and (33).

It is interesting to note here that the expression (35) for the drift velocity can be transformed to obtain the slip ratio v_g/v_f under the simplifying assumption that the average liquid velocity is much smaller than the vapor velocity. Then we have

$$\frac{v_g}{v_f} = \sqrt{\frac{\rho_f}{\rho_g}}\left[\frac{\sqrt{\alpha}}{1+75(1-\alpha)}\right]^{1/2} \quad (36)$$

for an annular flow in a pipe for which $\xi = \sqrt{\alpha}$. The above expression for slip ratio is similar to that obtained by Fauske (1962), namely, $v_g/v_f = (\rho_f/\rho_g)^{1/2}$, which has no dependence on the void fraction. The factor which takes

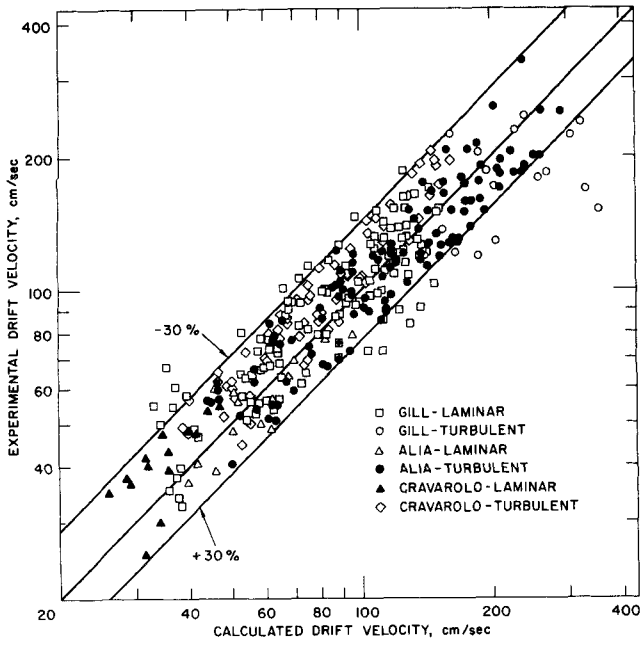

Fig. 3. Comparison of the theory to the experimental data.

the void fraction into account in Equation (36) varies roughly from 0.24 to 1 for the range of α given by $0.8 < \alpha < 1$. Therefore, it is expected that for a turbulent film the Fauske correlation will give reasonably accurate results at high void fractions.

COMPARISON WITH DATA AND DISCUSSIONS

Some experimental data for annular two-phase flow are available in the literature. One of the most extensive and reliable sets of data were obtained at Harwell by Hewitt and his associates. In particular, those obtained by Gill and Hewitt (1962) for upward concurrent flow of air-water mixtures at near atmospheric pressure in a 3.175 cm diam. acrylic resin tube have been used to verify the present correlation. Since the annular or drop-annular flow regime is confirmed for all the data points by their observations, all of their tabulated data (approximately 130 data points) have been used in the present study.

A second set of data used was that of Alia et al. (1965). These data were taken in 1.5 and 2.5 cm diam. tubes with water or ethyl alcohol as a liquid phase and argon as a gas phase. The ranges of the gas and liquid flow rates were 15 to 100 and 20 to 200 g/cm^2-s, respectively. The experiments were performed at near room temperature with system pressures up to 22 kg/cm^2. The total number of the data points was approximately 300; however, these data were taken at various flow regimes, and unfortunately there were no explicit indications of the flow regimes for any data point. Consequently, we have made flow regime analyses based on the flow regime map of Alia et al. (1965) as well as the method described in Collier (1972). These analyses show that only a limited number of data occurred for a pure annular flow regime. Approximately fifty points belong to the churn, slug, or bubbly flow regimes, and another fifty were in the transition region between the slug type and the annular or drop annular flow regimes. Thus, only 200 data points belonged to annular or drop annular flow regimes with moderate to high droplet entrainment. In order to show that the present model for V_{gj} can also be used for annular flow with moderated entrainment, approximately half of the data for the drop annular flow regime obtained with lower gas flow rates has been used together with the data for the pure annular flow regime. Consequently, about 125 data points from Alia et al. (1965) have been used in the present study.

Another set of data considered were those of Cravarolo et al. (1964) obtained under conditions very similar to those of Alia et al. (1965). In the experiment of Cravarolo, the void fractions were not measured directly but has been deduced from the pressure drop and wall shear stress measurements. As explained in Alia et al. (1965), this method of obtaining the void fraction is quite unreliable at high flow rates. Consequently, we eliminated some of the data based on the criteria given in Alia et al. (1965). Among the total number of 220 data points of Cravarolo, 180 were clearly in the drop annular flow regime, based on the given flow regime map and the flow regime criteria given in Collier (1972). Among these, about 90 data points have been eliminated because of the uncertainty of the void fraction prediction at high flow rates and the expected effect of higher entrainment.

The three sets of data considered constitute about 350 data points for annular and drop annular flow regimes. The comparison of the theoretical vapor drift velocities based on the present analysis [Equations (22) and (26) which are identical to Equations (28), (30), (32), and (33)] to the experimental data is given in Figure 3. Theoretical predictions are within about $\pm 30\%$ of the experimental values over a wide range of vapor drift velocity lying between 20 and 250 cm/s. A quite uniform distribution of the data can be attributed to the uncertainty in the determination of void fraction in the above experiments. We note here that these data include a considerable number of points taken in the drop annular flow regime at moderate gas flow rates. This indicates that the present vapor-drift-velocity correlation can be used both for ideal annular flows without entrainment as well as for drop annular flows with moderate to low entrainment. However, as the amount of liquid entrained in the gas core becomes large at high gas flow rates (Wallis, 1969; Ishii and Grolmes, 1975), the measured vapor drift velocity starts to depart considerably from the predicted values, and the present analysis overpredicts these values. The prediction of V_{gj} in the region where droplet entrainment is sufficiently high is beyond the scope of the present study.

The present constitutive equation for the vapor drift velocity in an annular flow regime has been obtained from the steady state and adiabatic formulation. The effects of heat transfer and phase changes on the vapor drift velocity were considered as secondary. These effects appear only indirectly through the local variables, such as the void fraction and the mixture velocity in the drift constitutive equation. It is a common practice to apply constitutive relationships obtained under steady state conditions to the transient problems, with an assumption that the parameters entering into a constitutive relationship are local variables and are functions of time. Therefore, the application of the present constitutive equation for V_{gj} to a transient two-phase annular flow with a phase change will be consistent with the common practice.

However, it should be noted that the basic assumption of the drift flux model is that there exists a strong coupling between the motions of two phases. Therefore, certain two-phase problems involving a sudden acceleration of one phase may not be appropriately described by this model. In these cases, inertia terms of each phase should be considered separately, that is, by use of two-fluid model. However, the real usefulness of the drift flux model in many practical engineering problems comes from the fact that even two-phase mixtures which are weakly coupled locally are strongly coupled when considered as a total system. This is because the relatively large axial extent of the systems usually gives sufficient interaction times for the momentum exchange between two phases.

ACKNOWLEDGMENT

The authors are grateful to A. Glassner for editing the manuscript and to George Hauser for his contributions in preparing programs for analyzing experimental data. This work was performed under the auspices of the U. S. Nuclear Regulatory Commission.

NOTATION

A = total flow area per channel
a, b, c = constants defined by Equation (31)
D = hydraulic diameter
f_i = interfacial friction factor
f_{wf} = wall friction factor based on film
g = acceleration of gravity
j = total volumetric flux
j_f = liquid volumetric flux, superficial liquid velocity
j_{ftr} = absolute liquid volumetric flux at laminar turbulent transition, defined by Equation (25)
p = pressure
P_i = wetted perimeter of interface
P_{wf} = wall perimeter wetted by film
Re_f = film Reynolds number defined by Equation (18)
t = time
V_{gj} = vapor drift velocity
v_f = average liquid velocity
v_g = average gas velocity
v_m = mixture center-of-mass velocity
v_r = $v_g - v_f$, relative velocity between phases
z = axial coordinate

Greek Letters

α = void fraction of gas phase
Γ_g = vapor generation rate
$\Delta\rho$ = density difference
δ = average film thickness
ϵ = roughness parameter of interface
μ_f = liquid viscosity
ξ = ratio of the two wetted perimeters given by Equation (14)
ρ_f = liquid density
ρ_g = gas density
ρ_m = mixture density
τ_i = interfacial shear stress
τ_{wf} = wall shear stress

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Manuscript received June 23, 1975; revision November 17 and accepted November 21, 1975.

Drop Size Distributions and Coalescence Frequencies of Liquid-Liquid Dispersions in Flow Vessels

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A flash photomicrographic method and modified dye-light transmittance technique are employed to measure size distributions and mixing frequencies in a turbulently agitated flow vessel. This liquid-liquid dispersion is nearly spatially homogeneous. Mixing frequencies are strongly dependent on impeller speed and moderately dependent on holdup fraction.

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